

$$\min_x x^2 \quad \text{s.t. } x \geq 1$$

1. Write down Lagrangian

$$L(x, \lambda) = x^2 - \lambda(x - 1)$$

2. Find optimal primal parameters $x^* = f(\lambda)$

$$x^* = \operatorname{argmin}_x L(x, \lambda) = f(\lambda)$$

$$\frac{\partial}{\partial x} L(x, \lambda) = 2x - \lambda = 0 \rightarrow x^*(\lambda) = \lambda/2$$

3. Plug back in Lagrangian to get dual

$$g(\lambda) = L(x^*(\lambda), \lambda)$$

$$g(\lambda) = \lambda^2/4 - \lambda(\lambda/2 - 1) = -\lambda^2/4 + \lambda$$

4. Find optimal dual parameters

5. Compute primal solution